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# Enhanced conformal $BMS_3$ symmetries

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Abstract: An enhanced version of the conformal $BMS_3$ algebra is presented. It is shown to emerge from the asymptotic structure of an extension of conformal gravity in 3D by Pope and Townsend that consistently accommodates an additional spin-2 field, once it is endowed with a suitable set of boundary conditions. The canonical generators of the asymptotic symmetries then span a precise nonlinear $W_{(2,2,2,2,1,1,1)}$ algebra, whose central extensions and coefficients of the nonlinear terms are completely determined by the central charge of the Virasoro subalgebra. The wedge algebra corresponds to the conformal group in four dimensions $SO(4,2)$ and therefore, enhanced conformal $BMS_3$ can also be regarded as an infinite-dimensional nonlinear extension of the $AdS_5$ algebra with nontrivial central extensions. It is worth mentioning that our boundary conditions might be considered as a starting point in order to consistently incorporate either a finite or an infinite number of conformal higher spin fields.

Keywords: Conformal and W Symmetry, Higher Spin Symmetry, Gauge-Gravity Correspondence, Classical Theories of Gravity

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## 1 Introduction

Finding a bona fide conformal extension of the BMS algebra appears to be a hard nut to crack (see e.g. [1–5]). Nevertheless in three spacetime dimensions the task can be successfully achieved, provided that an infinite number of superdilatations and superspecial conformal transformations are incorporated within a nonlinear algebra [6]. Specifically, the commutator of supertranslations with superspecial conformal transformations acquires quadratic and cubic terms made of superrotations and superdilatations. The conformal $BMS_3$ algebra has been shown to emerge in different physical setups, as it is the case of the asymptotic symmetry algebra of conformal gravity in 3D [6], as well as from the free field realization of the $BMS_3$ Ising model in 2D [7]. Further related results can be found in [8, 9].

The conformal $BMS_3$ algebra seems to be very rigid, since the central extensions and the coefficients of the nonlinear terms become entirely determined by the central charge of the Virasoro subalgebra. Indeed, the conditions obtained from the Jacobi identity turn out to be very stringent, which suggests that the algebra is unique. In this sense, since the conformal $BMS_3$ algebra looks undeformable, one may wonder whether it might be enhanced in some appropriate way. As a strategy to achieve this task we propose exploring the asymptotic structure of a suitable extension of conformal gravity in 3D [10, 11]. A nice and simple theory enjoying the sought features was proposed long ago by Pope and Townsend [12] with the aim of further enlarging it in order to describe an infinite tower of conformal higher spin fields in 3D, along the lines of [13]. More recently, conformal gravity was shown to admit a different extension that accommodates a large class of theories with a finite number of conformal higher spin fields [14].[1]

The theory proposed in [12] describes a non-gauged spin-2 field consistently coupled to conformal gravity, and it can also be formulated in terms of a Chern-Simons action for $so(4,2)$, after a suitable gauge choice akin to that of Horne and Witten for the case of pure conformal gravity [11].

In the next section we show that the searched-for enhancement of the conformal $BMS_3$ algebra naturally emerges from the asymptotic structure of the extension of conformal gravity aforementioned.

[1] Additional interesting results concerning conformal higher spin fields in 3D can be found in e.g., [15–21].

## 2 Asymptotic structure of extended conformal gravity in 3D

Following Pope and Townsend [12] the three-dimensional conformal algebra $so(3,2)$, spanned by $\{J_a, P_a, K_a, D\}$ with $a = 0,1,2$, is enhanced to that of $so(4,2)$ by enlarging the set of generators to include $\{U_a, U, V\}$. This is also isomorphic to the algebra of depth-two conformal gravity in Grigoriev et al. [14].

For our purposes, it is convenient to arrange the generators in a different way. The generators of the $so(2,1) \approx sl(2,\mathbb{R})$ subalgebra that commutes with the Lorentz subalgebra (spanned by $J_a$) are given by $T^I = \{U, V, D\}$ with $I = 0,1,2$, so that the remaining generators $P_a^I = \{P_a, K_a, U_a\}$ also behave like vectors of $sl(2,\mathbb{R})$. The full $so(4,2)$ algebra then explicitly reads

$$\begin{aligned} [J_a, J_b] &= \epsilon_{abc} J^c \, ; \quad \left[J_a, P_b^I\right] = \epsilon_{ab}{}^c P_c^I \, , \\ \left[T^I, T^J\right] &= \epsilon^{IJK} T_K \, ; \quad \left[T^I, P_a^J\right] = \epsilon^{IJ}{}_K P_a^K \, , \\ \left[J_a, T^I\right] &= 0 \, ; \quad \left[P_a^I, P_b^J\right] = -2\tau^{IJ} \epsilon_{abc} J^c - 2\eta_{ab} \epsilon^{IJK} T_K \, , \end{aligned} \tag{2.1}$$

where both $\eta_{ab}$ and $\tau_{IJ}$ stand for the flat Minkowski metric in 3D. It is useful to express $\tau^{IJ}$ in light cone coordinates ($\tau^{01} = \tau^{10} = \tau^{22} = 1$) so that Poincaré translations and special conformal transformations correspond to $P_a = P_a^0$ and $K_a = P_a^1$, respectively; while dilations do for $D = T^2$.

We choose the normalization of the Cartan-Killing metric so that it reads

$$\langle J_a J_b \rangle = \eta_{ab} \, ; \quad \left\langle P_a^I P_b^J \right\rangle = -2\eta_{ab} \tau^{IJ} \, ; \quad \langle T_I T_J \rangle = \tau_{IJ} \, , \tag{2.2}$$

and hence, the extension of conformal gravity in 3D can be expressed in terms of a Chern-Simons action

$$I[A] = \frac{k}{4\pi} \int \left\langle A dA + \frac{2}{3} A^3 \right\rangle \, , \tag{2.3}$$

for a gauge field given by

$$A = \omega^a J_a + E_I^a P_a^I + M_I T^I \, , \tag{2.4}$$

where $e^a = E_0^a$ and $\omega^a$ stand for the dreibein and the dualized spin connection, while $s^a = E_2^a$ corresponds to the one-form associated to the spin-2 gauge field [14].[2]

### 2.1 Boundary conditions and enhanced conformal $\mathrm{BMS}_3$ algebra

Following the lines of [22], a gauge choice of the form $A = g^{-1} a g + g^{-1} dg$, with a suitable group element $g = g(r)$, allows to completely gauge away the radial dependence of the asymptotic form of the connection, so that the remaining analysis can be readily performed in terms of the auxiliary gauge field $a = a_t dt + a_\varphi d\varphi$ that depends only on time and the angular coordinate.

We then propose the following fall-off of the gauge field

$$a = \left[ J_1 + \frac{2\pi}{k} \left( \mathcal{J} + \frac{2\pi}{k} \Lambda_{(2)} \right) J_0 + \frac{\pi}{k} \mathcal{P}_I P_0^I + \frac{2\pi}{k} \mathcal{M}_I T^I \right] (d\varphi + dt) \, , \tag{2.5}$$

[2]This field was initially identified as a spin-2 non-gauge field [12].



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where $\Lambda_{(2)} = \tau_{IJ}\Lambda^{IJ}_{(2)}$, with

$$\Lambda^{IJ}_{(2)} = \frac{1}{4}\left(\mathcal{M}^I\mathcal{M}^J - \tau^{IJ}\mathcal{M}^K\mathcal{M}_K\right) , \tag{2.6}$$

and the dynamical fields $\mathcal{J}$, $\mathcal{P}_I$, $\mathcal{M}_J$ depend only on $t$, $\varphi$.

The asymptotic behavior is preserved under gauge transformations $\delta a = d\Omega + [a, \Omega]$, with a Lie-algebra-valued parameter given by

$$\Omega\left[\epsilon, \zeta^I, \lambda^J\right] = \epsilon J_1 - \zeta_I P_1^I - \left(\lambda_I - \frac{2\pi}{k}\mathcal{M}_I\epsilon\right) T^I + \Theta\left[\epsilon, \zeta^I, \lambda^J\right] , \tag{2.7}$$

with

$$\begin{aligned}\Theta =& -\epsilon' J_2 + \frac{2\pi}{k}\left(\zeta'_K - \frac{2\pi}{k}\epsilon^{IJ}{}_K\zeta_I\mathcal{M}_J\right) P_2^K + \frac{2\pi}{k}\left[\epsilon\left(\mathcal{J} + \frac{2\pi}{k}\Lambda_{(2)}\right) + \zeta_I\mathcal{P}^I - \frac{k}{2\pi}\epsilon''\right] J_0 \\ & - \frac{2\pi}{k}\left[\zeta_K\left(\mathcal{J} + \frac{2\pi}{k}\Lambda_{(2)}\right) + \frac{8\pi}{k}\zeta_I\Lambda^I_{(2)K} + \epsilon^{IJ}{}_K\left(\zeta_I\mathcal{M}'_J + 2\zeta'_I\mathcal{M}_J\right) - \frac{1}{2}\epsilon\mathcal{P}_K - \frac{k}{2\pi}\zeta''_K\right] P_0^K ,\end{aligned} \tag{2.8}$$

depending on chiral functions of $t$, $\varphi$ fulfilling $\dot{\epsilon} = \epsilon'$, $\dot{\zeta}_I = \zeta'_I$, $\dot{\lambda}_I = \lambda'_I$. Note that anti-chiral functions would be obtained if the asymptotic behavior of the gauge field in (2.5) were chosen as a one-form along $d\varphi - dt$ (instead of $d\varphi + dt$).

The transformation law of the dynamical fields is then given by

$$\begin{aligned}\delta\mathcal{J} &= 2\mathcal{J}\epsilon' + \mathcal{J}'\epsilon - \frac{k}{2\pi}\epsilon''' + 2\mathcal{P}^I\zeta'_I + 2\mathcal{P}'_I\zeta^I - \mathcal{M}^I\lambda'_I , \\ \delta\mathcal{P}^I &= 2\mathcal{P}^I\epsilon' + \mathcal{P}^{I\prime}\epsilon - 4\left(\mathcal{J}\tau^{IJ} - \tilde{\Lambda}^{IJ}_{(2)}\right)\zeta'_J - 2\left[\left(\mathcal{J}\tau^{IJ} - \tilde{\Lambda}^{IJ}_{(2)}\right)' + \epsilon^{IJK}\mathcal{M}''_K + \frac{1}{2}\tilde{\Lambda}^{IJ}_{(3)}\right]\zeta_J \\ &\quad + 6\left(\epsilon^{IJK}\mathcal{M}_J\zeta'_K\right)' - \epsilon^{IJK}\mathcal{P}_J\lambda_K + \frac{k}{\pi}\zeta^{I\prime\prime\prime} , \\ \delta\mathcal{M}_I &= \mathcal{M}_I\epsilon' + \mathcal{M}'_I\epsilon + \epsilon_{IJK}\mathcal{P}^J\zeta^K - \epsilon_I{}^{JK}\mathcal{M}_J\lambda_K - \frac{k}{2\pi}\lambda'_I .\end{aligned} \tag{2.9}$$

with $\tilde{\Lambda}^{IJ}_{(2)}$ and $\tilde{\Lambda}^{IJ}_{(3)}$ being symmetric and antisymmetric in $I$, $J$, respectively, and defined as

$$\begin{aligned}\tilde{\Lambda}^{IJ}_{(2)} &= -\frac{2\pi}{k}\left(\Lambda_{(2)}\tau^{IJ} + 6\Lambda^{IJ}_{(2)}\right) , \\ \tilde{\Lambda}^{IJ}_{(3)} &= -\frac{4\pi}{k}\left[2\left(\mathcal{J} + \frac{4\pi}{k}\Lambda_{(2)}\right)\epsilon^{IJK}\mathcal{M}_K + \mathcal{M}^{[I}\mathcal{M}'^{J]}\right] .\end{aligned} \tag{2.10}$$

The generators of the asymptotic symmetries can then be straightforwardly obtained following different approaches, as in [23, 24] (see also e.g., [25–27]), and they are given by

$$\mathcal{Q}\left[\epsilon, \zeta_I, \lambda_I\right] = -\int\left(\epsilon\mathcal{J} + \zeta_I\mathcal{P}^I - \lambda_I\mathcal{M}^I\right) d\varphi , \tag{2.11}$$

so that their algebra can be extracted from their Dirac brackets; or in a more direct way, by virtue of $\delta_{\eta_1}\mathcal{Q}\left[\eta_2\right] = \left\{\mathcal{Q}\left[\eta_2\right], \mathcal{Q}\left[\eta_1\right]\right\}$, and the transformation law of the dynamical fields in (2.9).



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The algebra of the asymptotic symmetry generators is then found to be described by

$$
\begin{aligned}
\{\mathcal{J}(\phi),\mathcal{J}(\varphi)\} &= -2\mathcal{J}(\phi)\delta'(\phi-\varphi)-\delta(\phi-\varphi)\,\mathcal{J}'(\phi)+\frac{k}{2\pi}\delta'''(\phi-\varphi)\,,\\
\left\{\mathcal{J}(\phi),\mathcal{P}^K(\varphi)\right\} &= -2\mathcal{P}^K(\phi)\delta'(\phi-\varphi)-\delta(\phi-\varphi)\,\mathcal{P}^{K\prime}(\phi)\,,\\
\left\{\mathcal{J}(\phi),\mathcal{M}^I(\varphi)\right\} &= -\mathcal{M}^I(\phi)\delta'(\phi-\varphi)\,,\\
\left\{\mathcal{M}^I(\phi),\mathcal{M}^J(\varphi)\right\} &= \epsilon^{IJK}\mathcal{M}_K\delta(\phi-\varphi)-\frac{k}{2\pi}\tau^{IJ}\delta'(\phi-\varphi)\,,\\
\{\mathcal{P}_I(\phi),\mathcal{M}_J(\varphi)\} &= \epsilon_{IJK}\mathcal{P}^K(\phi)\delta(\phi-\varphi)\,,\\
\left\{\mathcal{P}^I(\phi),\mathcal{P}^J(\varphi)\right\} &= 4\left(\mathcal{J}(\phi)\tau^{IJ}-\tilde{\Lambda}^{IJ}_{(2)}(\phi)\right)\delta'(\phi-\varphi)\\
&\quad+2\left(\mathcal{J}(\phi)\tau^{IJ}-\tilde{\Lambda}^{IJ}_{(2)}(\phi)+\epsilon^{IJK}\mathcal{M}'_K(\phi)\right)'\delta(\phi-\varphi)\\
&\quad-6\left(\epsilon^{IJK}\mathcal{M}_K(\phi)\delta'(\phi-\varphi)\right)'+\tilde{\Lambda}^{IJ}_{(3)}(\phi)\delta(\phi-\varphi)-\frac{k}{\pi}\tau^{IJ}\delta'''(\phi-\varphi)\,.
\end{aligned}
\tag{2.12}
$$

Expanding in Fourier modes according to $X=\frac{1}{2\pi}\sum_m X_m e^{im\varphi}$, the algebra reads

$$
\begin{aligned}
i\{\mathcal{J}_m,\mathcal{J}_n\} &= (m-n)\,\mathcal{J}_{m+n}+m\left(m^2-1\right)k\delta_{m+n,0}\,,\\
i\left\{\mathcal{J}_m,\mathcal{P}^I_n\right\} &= (m-n)\,\mathcal{P}^I_{m+n}\,,\\
i\left\{\mathcal{J}_m,\mathcal{M}^I_n\right\} &= -n\mathcal{M}^I_{m+n}\,,\\
i\left\{\mathcal{M}^I_m,\mathcal{M}^J_n\right\} &= i\epsilon^{IJ}{}_K\mathcal{M}^K_{m+n}+k\tau^{IJ}m\delta_{m+n,0}\,,\\
i\left\{\mathcal{P}^I_m,\mathcal{M}^J_n\right\} &= i\epsilon^{IJ}{}_K\mathcal{P}^K_{m+n}\,,\\
i\left\{\mathcal{P}^I_m,\mathcal{P}^J_n\right\} &= -2\,(m-n)\,\mathcal{J}_{m+n}\tau^{IJ}+(m-n)\,\tilde{\Lambda}^{IJ}_{(2)m+n}\\
&\quad-2i\left(m^2-mn+n^2-1\right)\epsilon^{IJ}{}_Q\mathcal{M}^Q_{m+n}+\tilde{\Lambda}^{IJ}_{(3)m+n}+2k\tau^{IJ}m\left(m^2-1\right)\delta_{m+n,0}\,,
\end{aligned}
\tag{2.13}
$$

where the zero mode of $\mathcal{J}_n$ has been shifted as $\mathcal{J}_0\to\mathcal{J}_0-\frac{k}{4\pi}$, and the nonlinear terms given by

$$
\begin{aligned}
\tilde{\Lambda}^{IJ}_{(2)m} &= \frac{4}{k}\tau^{IJ}\tau_{KL}\sum_n\mathcal{M}^K_{m-n}\mathcal{M}^L_n-\frac{3}{k}\sum_n\mathcal{M}^I_{m-n}\mathcal{M}^J_n\,,\\
\tilde{\Lambda}^{IJ}_{(3)m} &= -\frac{4}{k}\epsilon^{IJ}{}_K\sum_p\left(\mathcal{J}_{m+p}-\frac{1}{k}\tau_{QR}\sum_n\mathcal{M}^Q_{m-n-p}\mathcal{M}^R_n\right)\mathcal{M}^K_p+\frac{i}{2k}\sum_n n\mathcal{M}^{[I}_n\mathcal{M}^{J]}_{m-n}\,,
\end{aligned}
\tag{2.14}
$$

possess (anomalous) conformal weight 2 and 3, respectively. Indeed, the conformal weight of the generators $\mathcal{J}_m$, $\mathcal{P}^I_m$ is 2, while that of the currents $\mathcal{M}^I_m$ is clearly 1.

The wedge algebra is then given by that of the original gauge group SO(4, 2) in (2.1), being recovered once the nonlinear terms are dropped and the modes are restricted according to $|m|<s$, where $s$ stands for the conformal weight of the generators, followed by a simple change of basis.

Note that since the Lorentz subalgebra is non-principally embedded within the wedge algebra, according to the conformal weight of the generators, the enhanced conformal $\mathrm{BMS}_3$ algebra (2.13) can be regarded as a $W_{(2,2,2,2,1,1,1)}$ algebra (see e.g. [28, 29]).

It is also worth highlighting that the Virasoro central charge gives support to the nonlinear terms, and therefore, the enhanced conformal $\mathrm{BMS}_3$ algebra turns out to be well-defined



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provided the central charge does not vanish. Nonetheless, for the quantum algebra this is not necessarily the case because the central extensions and the coefficient in front of the nonlinear terms generically acquire corrections.

## 3 Ending remarks

The enhanced conformal $BMS_3$ algebra (2.13) inherits the "rigidity" of its non enhanced version in [6], since all of the central extensions and the coefficients of the nonlinear terms also become entirely fixed in terms of the central charge of the Virasoro subalgebra, determined by the Chern-Simons level $k$. This can be traced back by the fact that the extension of the conformal algebra $so(4,2)$ is semisimple, so that it possesses a unique invariant bilinear form given by the Cartan-Killing metric that can be normalized as in (2.2).

It must be stressed that supertranslations no longer commute with themselves in the enhanced version of the conformal $BMS_3$ algebra, and this is also the case for the superspecial conformal transformations. Indeed, from the corresponding commutator in (2.13), one can read that

$$i\left\{\mathcal{P}_m,\mathcal{P}_n\right\} = i\left\{\mathcal{P}^0_m,\mathcal{P}^0_n\right\} = -\frac{3}{k}\left(m-n\right)\sum_p \mathcal{M}^0_{m+n-p}\mathcal{M}^0_p\,, \tag{3.1}$$

$$i\left\{\mathcal{K}_m,\mathcal{K}_n\right\} = i\left\{\mathcal{P}^1_m,\mathcal{P}^1_n\right\} = -\frac{3}{k}\left(m-n\right)\sum_p \mathcal{M}^1_{m+n-p}\mathcal{M}^1_p\,, \tag{3.2}$$

and hence, commutativity is lost due to nonlinear contributions of the current generators even at the classical level, being clearly persistent in the quantum realization. This is in stark contrast with what occurs for the (non enhanced) conformal $BMS_3$ algebra in [6], since commutativity holds in that case. Indeed, $BMS_3$ is a subalgebra of its conformal extension; nevertheless, it is not a subalgebra of its enhanced conformal extension due to the nonlinear terms in the currents.

It is worth noting that the enhanced conformal $BMS_3$ algebra (2.13) can also be seen as an infinite-dimensional nonlinear extension of the $AdS_5$ algebra with nontrivial central charges.[3] Thus, the obstruction to include non trivial central extensions for semisimple algebras, supported by a classical theorem of algebraic cohomology (see e.g. [30]), can be circumvented due to the nonlinearity of the algebra.

It is also interesting to explore whether the black hole solutions of conformal gravity in 3D [21, 33–35] could be endowed with an additional spin-2 field in the context of the extension of conformal gravity of Pope and Townsend [12] and Grigoriev et al. [14]. In order to suitably explore their properties, the asymptotic behavior discussed here should be extended along the lines of [36, 37] so as to include the chemical potentials that correspond to the enlarged set of global charges in (2.11).

As a final remark, it is worth exploring whether the fall-off of the gauge fields implemented by our boundary conditions could be suitably extended to incorporate either a finite or an infinite number of conformal higher spin fields along the lines of [14] and [12], respectively. It is then natural to expect that the full extension of the $BMS_3$ algebra that would emerge from such

[3] An infinite-dimensional linear extension of $AdS_5$ has been proposed in [31, 32].

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scenarios should necessarily be nonlinear in a two-folded way. Indeed, nonlinear extensions of $BMS_3$ algebra are known to appear not only for its conformal enhancement, but also from the presence of bosonic or fermionic higher spin fields as in [38–41] and [42, 43], respectively.


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